Data & Knowledge Engineering, Vol. 19, No. 1(1996), pp. 39-64

Analysis of Binary/Ternary Cardinality Combinations in Entity-Relationship Modeling

Trevor H. Jones & Il-Yeol Song

College of Information Studies
Drexel University
Philadelphia, PA., 19104.

Abstract

In this paper, we discuss the simultaneous existence, and relationships, between binary and ternary relationships in entity-relationship (ER) modeling. We define the various interpretations that can be applied to the simultaneous existence of ternary and binary relationships having the same participating entities. We have identified that only certain cardinalities are permitted to exist simultaneously in such ER structures. We demonstrate which binary relationship cardinalities are permitted within ternary relationships, during ER modeling. We develop an Implicit Binary Cardinality (IBC) rule, which states that, in any ternary relationship, the cardinality of any binary relationship embedded in the ternary, is many-to-many when there are no explicit constraints on the data instances. We then present an Explicit Binary Permission (EBP) rule, which explains and enumerates all permitted binary relationships for various cardinalities of ternary relationships. Finally we present an Implicit Binary Override (IBO) rule, which states that the implicit binary cardinalities can be constrained in a ternary relationship by an explicitly imposed binary relationship. We then use these rules to consider the further implicit dynamics of ternary relationships when multiple binary relationships are imposed.

In discussing these findings, we consider the rules in the context of supporting functional dependency analysis. The relevance of the findings is presented in the context of ensuring that all functional dependencies associated with ternary relationships are correctly applied and identifying the potential for decomposing the ternary relationship into multiple binary relationships based on all explicit functional dependencies.

KEYWORDS: Entity-relationship model, binary relationship, ternary relationship, ternary decomposition, data modeling, binary imposition rules.

1. INTRODUCTION

One of the most difficult problems in using entity relationship (ER) models (Chen, 1976) for database design is the decision of whether to use a ternary or multiple binary relationships in complex entity relationship types. Although the use of ternary relationships is widely accepted in ER modeling, almost no formal analysis of ternary relationships has been done. This has led to incomplete knowledge regarding ternary relationships and results in no formalized alternative to the currently accepted methods for dealing with ternary relationships, namely a binary *conversion* of the ternary structure. In order to establish criteria for the decomposition of ternary relationships into multiple binary relationships, we investigate not only the implicit structure and inter-entity relationships, but also which binary relationship cardinalities can be permitted in ternary relationships.

In this paper we present a comprehensive set of rules, establishing the relationships between variously configured ternary relationships and binary relationships embedded within those ternary relationships. We believe these rules can play an important role in considering the decomposition of ternary relationships into multiple binary relationships. One of our goals is to provide a set of easy to use rules for ternary relationships, by establishing the connections between relational theory and semantic modeling as they relate to ternary structures.

Research on the relationships between ternary and binary configurations in ER modeling is scarce. Chung, Nakamura and Chen (1982) discuss the decomposition of an entity-relationship diagram (ERD) into 4th normal form (4NF). Ling (1985a) discusses the idea of normalization of ERD's, and further investigates the relationship between ERD's, multivalued dependencies and join dependencies (Ling, 1985b). Jajodia et al. (1983) discuss the equivalence of ERD's. Teorey et al. (1986) states that a ternary relationship type should be used only when the association among three entity types cannot be represented by several binary relationships, but does not discuss specifically when this can occur, or if several binary relationships have an equivalent informational capacity.

None of these works clearly shows the relationship between binary and ternary representations. Some authors, e.g., Hawryszkiewycz (1991), offer solutions to *translating* a ternary relationship into a set of binary relationships. It is emphasized that this is a conversion, rather than decompositional, approach; that is, once the ternary relationship has been established, this technique can be used to translate the ERD into a binary representation of the ERD. The conversion provides only for a generic methodology for creating a relation (gerund) from the ternary relationship. In the conversion approach, the ternary relationship is converted into a new entity type, and a binary relationship is created between the new converted entity type and each participating entity type, where the cardinality of those binary relationships is M:1. In the decompositional approach, the ternary relationship is removed. It is replaced with 2 or 3 binary relationships between the participating entity types . These binary relationship replacements are based on the complete set of functional dependencies implicit to the original ternary relationship.

The conversion process drastically changes the nature of a data modeling construct from a relationship to an entity. This change is purely artificial. The change is required due to the limitations of the modeling constructs which do not allow ternary relationships. The decompositional approach seeks to identify a minimal number of binary relationships which losslessly replace a ternary relationship.

Before any alteration of the modeled structure, we need to decide whether to model the situation using a ternary or a set of binary relationships. Elmasri and Navathe (1989) provide a single guideline saying that a ternary relationship can be decomposed if it contains an embedded binary relationship with a 1:1 cardinality. They do not clarify the theoretical existence of 1:1 cardinalities within ternary relationships. Teorey (1990) identifies when a ternary relation is BCNF or 4NF, using the notion of functional dependencies and multivalued dependencies. Again, however, Teorey does not consider the dynamics when ternary and binary relationships co-exist with the same participating entities.

One of the motivations behind our work is to clarify the validity of ERD structures containing both a ternary relationship and its associated binary relationships. For example, suppose we have an ERD as shown in Figure 1. The structural representation of Figure 1 has been used several times, in previously published works (Jajodia et al. (1983), Markowitz & Shoshani (1989), Markowitz & Shoshani (1992)), as a *valid* example and *basis* for other arguments. How do we know whether the 1:M cardinality of the D_P relationship is valid in the context of the M:N:1 ternary relationship? In this paper we establish rules to determine that such an imposition is in fact invalid (see Section 4.1, Table 7), if D_P is semantically a subset of E_D_P relationship.

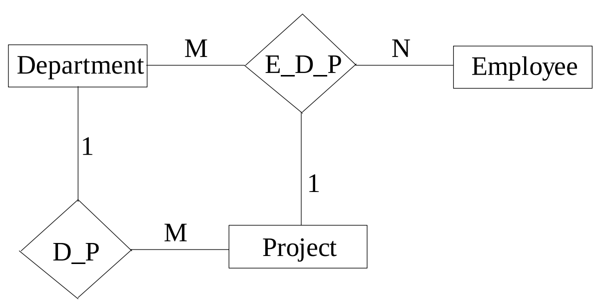


Figure 1. M:N:1 Ternary Relationship with
Imposed 1:M Binary Relationship

The use of such invalid models, indicate a continuing lack of understanding with respect to the use of ternary relationships in ER modeling. Additionally there is an absence of research concerning the implicit dynamics of ternary and binary relationship combinations.

We believe our paper is the first to address the issues of embedded binary combinations in ternary relationships; which combinations are permitted; and how these combinations address the decomposition of ternary relationships into multiple binary relationships.

For the sake of uniformity in presentation, we briefly define our terminology used throughout this paper:

Cardinality Constraint: a cardinality constraint assists in describing a relationship between two or more entities. It constrains the maximum number of potential relationships that an entity can participate in.

Binary Relationship: a binary relationship is a relationship of degree two. That is the relationship contains two participating entities. It can take the form of 1:1, 1:M, or M:N.

Ternary Relationship: a ternary relationship is a relationship of degree three. That is, a relationship that contains three participating entities. Cardinalities for ternary relationships can take the form of 1:1:1, 1:1:M, 1:M:N or M:N:P. The cardinality constraint of an entity in a ternary relationship is defined by a pair of two entity instances associated with the other single entity instance. For example, in a ternary relationship R(X, Y, Z) of cardinality M:N:1, for each pair of (X, Y) there is only one instance of Z; for each pair of (X, Z) there are N instances of Y; for each pair of (Y, Z) there are M instances of X.. A cardinality of 1 also indicates a relational unique key of the two attributes which are opposite the 1 cardinality. Therefore for a relation R(X,Y,Z) with cardinality of 1:1:M respectively, the composite (Y,Z) and the composite (X,Z) are candidate keys.

Decomposition: Consider a relation R(X, Y, Z) where X, Y and Z can individually be a group of attributes. Then R(X, Y, Z) can be losslessly decomposed into R1(X, Y) and R2(X, Z) if

R(X, Y, Z) = R1(X, Y) ∞ R2(X, Z)   where ∞ represents the natural join operator.

A lossless decomposition occurs if the natural join of the two subset relations created by the

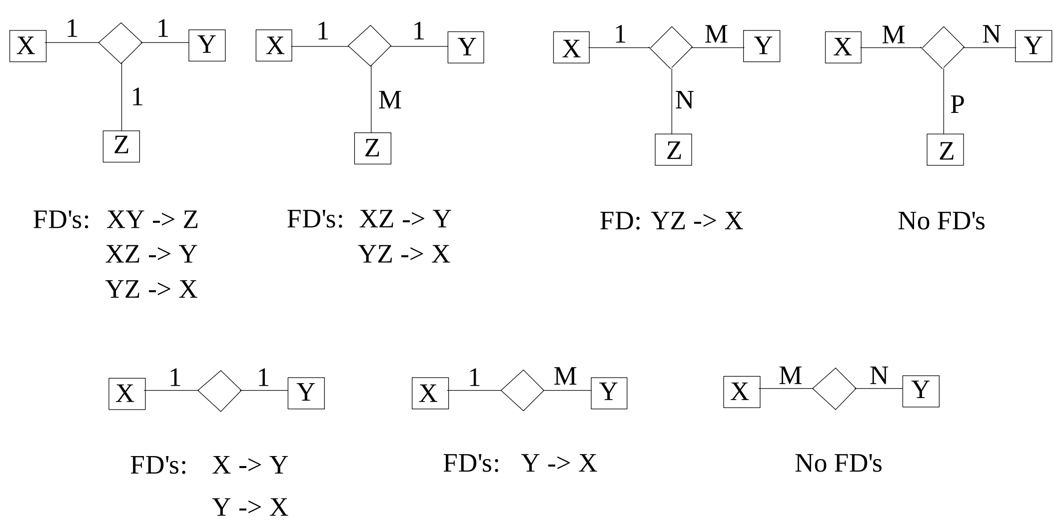


Figure 2. Functional dependencies for all ternary/binary cardinalities

decomposition, results in the original relation, without additional or missing tuples.

Functional Dependency: the functional dependencies that can be derived from the various cardinalities of binary and ternary relationships are detailed in Teorey (1990) and as shown in Figure 2.

The remainder of the paper is organized as follows: Section 2 discusses the domain of our arguments. It identifies the various interpretations that can be applied to ternary and binary combinations and specifies the interpretation from which we draw our work. Section 3 investigates the potential binary relationships that are implicit to ternary relationships of the various cardinalities, given no explicit constraints. We establish the first of three rules, the Implicit Binary Cardinality(IBC) rule. In Section 4, we develop the Explicit Binary Permission(EBP) rule, and the Implicit Binary Override(IBO) rule, following the analysis of imposing a single binary relationship on a ternary relationship. Section 5 builds on the single imposition analysis of Section 4, and enumerates the results of imposing multiple binary relationships on a single ternary relationship. Section 6 considers the ramifications of our findings on potential decomposition strategies for ternary relationships and the relevance to ER modeling in general. Section 7 summarizes our paper.

2. SEMANTIC CONTEXT

In Entity-Relationship (ER) modeling, the use of ternary and binary relationships indicate a specific semantic relationship between the participating entities. Consider a ternary relationship R(X, Y, Z) of cardinality 1:1:1. This relationship indicates that for each pair (X, Y) there is only one instance of Z; for each pair (X, Z) there is only one instance of Y; for each pair (Y, Z) there is only one instance of X. Semantically, then, this relationship designation forces us to consider all three entities with each other, simultaneously. Considering the relationship involving only two entities without reference to the third may, or may not, conform to the semantics of the ternary relationship. Within the same semantic requirements of the ternary, it may be possible that some form of explicit constraint exists between one or more pairs of the participating entities; that is, a subset of the ternary entities is constrained to a particular relationship cardinality. In this case the binary relationship is semantically a subset of the ternary relationship and constrains the instances of the ternary relationship. We term this type of binary relationship a **Semantically Constraining Binary (SCB) Relationship.** This type of SCB relationship is the focus of our paper.

Alternatively, the three participating entities in the ternary relationship, may have one or more binary relationships, between each other that are semantically unrelated to the ternary relationship itself. We call these relationships, **Semantically Unrelated Binary (SUB) Relationships**. This type of SUB relationship is excluded from consideration in the context of our paper.

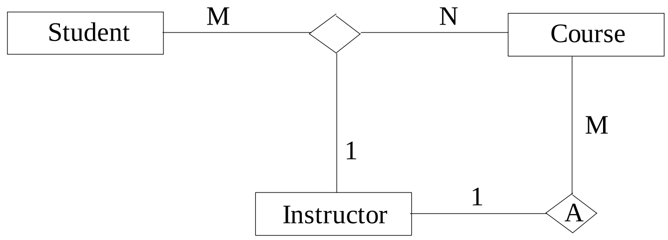


Figure 4. M:N:1 Ternary Relationship with
Simultaneous M:1 Binary Relationship

An example helps to clarify the semantic difference of the two points being made. Consider three entities Student, Course, Instructor. In an academic relationship we could model this as a ternary relationship of cardinality M:N:1 (Figure 3).

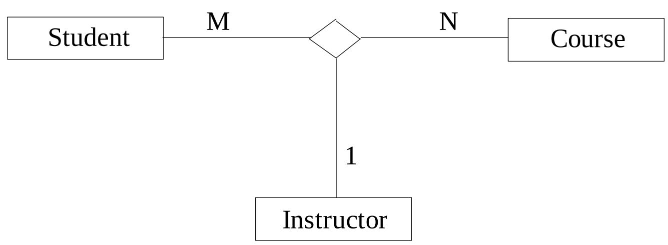


Figure 3. M:N:1 Ternary Relationship

This may provide a relation as follows:

	STUDENT	COURSE	INSTRUCTOR
1	Mike	Physics	Jones
2	Anne	Physics	Jones
3	Mike	Chemistry	Jones
4	Anne	Chemistry	Song

We note in this relation, which conforms to the cardinality of M:N:1, the relationship between each *pair* of entities is M:N. If we impose an external constraint on the ternary that *each course can only be taught by a single instructor*, then obviously tuple 3 or 4 is disallowed. We have placed an explicit constraint on the relation restricting certain instance combinations, whilst maintaining the cardinality of the ternary relationship. This constraint could be modeled as shown in Figure 4.

If we now wish to add another relationship to the ERD that contains entities from the ternary, but where the new relationship is unrelated to the ternary, we must logically incorporate it into the ERD model. For example, along with the relationship shown in Figure 4 between instructor and course (A), we may also have relationships indicating *which courses instructors are capable of teaching* (B), and *which courses instructors have historically taught* (C). These additional relationships would be modeled as binary relationships of cardinality M:N. These new relationships would be added to the original ternary relationship from Figure 4, as shown in Figure 5.

Notice that the additional binary relationships (B and C) between instructor and course have their own existence apart from the semantics of the ternary relationship.

From Figures 4 and 5, one problem occurs with the diagrammatic notation. We have indicated that relationship (A) is a constraining, or SCB, relationship of the ternary. Relationships (B) and (C) are legitimate relationships existing separately from the ternary. They are SUB relationships. The diagrammatic notation used and generally adopted in ER modeling is not sufficient to distinguish the semantically different relationship types. We therefore introduce a broken line

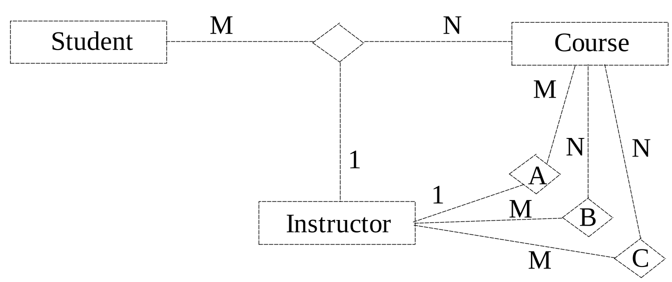


Figure 5. M:N:1 Ternary Relationship with Multiple
Simultaneous Binary Relationships

notation to signify the types of relationships discussed in this paper, which are the constraining or SCB relationships embedded in the ternary. Using this notation, Figure 5, above, would appear as shown in Figure 6.

We reiterate here that relationships of the types described as (B) and (C) above are separate and irrelevant to the discussion in this paper. We are interested in the potential combinations of

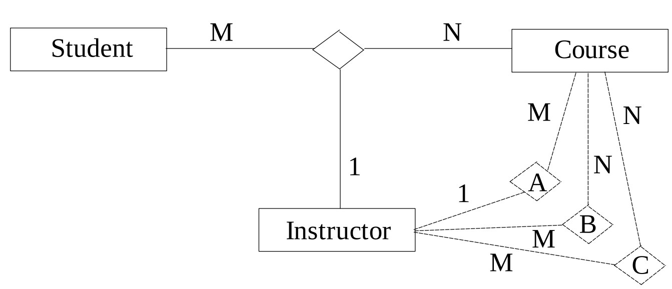


Figure 6. M:N:1 Ternary Relationship with SCB (A)
and SUB (B, C) Binary Relationships

binary and ternary instances within the constraints of the ternary itself, which by example are represented in Figure 6, relationship (A).

3. TERNARY -- BINARY COMBINATIONS

The investigation and discussion of ternary and binary relationships can be considered on two levels:

i. we can consider ternary relationships as sets of instances of three entities with the combination of each set being controlled by the specified cardinality and the number of instances of each entity.

ii. We must also consider that any specified ternary relationship will create multiple sets of binary relationships between all combinations of pairs.

Sections 3.1--3.4, consider the various ternary relationship cardinalities and investigates the binary relationships that can be developed within each cardinality, given no externally imposed restrictions.

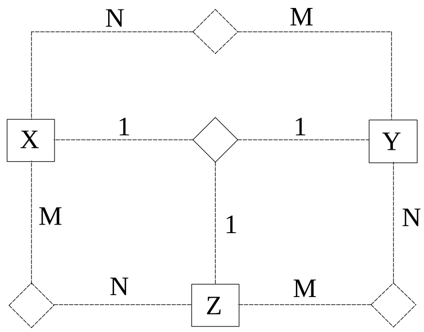
Cardinalities of ternary relationships can be considered in 4 categories (1:1:1, 1:1:M, 1:M:N and M:N:P). These are each considered separately below. In generic terms, we will consider three entities X, Y, and Z. Instances of each entity will be identified by notation following the entity identifier. (Therefore instances of entity X will appear X1, X2, ..., Xn and entity Z will appear Z1, Z2, ..., Zn, etc.) More realistic instances as examples are also used to show interpretations of the generic reasoning.

3.1. Ternary Cardinality -- 1:1:1

The 1:1:1 cardinality, in a ternary relationship, is interpreted as indicating a single instance of either X, Y or Z existing only with a single unique pair of instances of the other two entities. That is for a pair of (XY) instances, there is only one instance for Z; for a pair of (XZ) instances, there is only one instance for Y; for a pair of (YZ) instances, there is only one instance of X. Consider the example where each entity has a single instance; the ternary representation is (X1,Y1,Z1). No other combinations of the identified entity instances are possible and this combination of instances satisfies the cardinality requirements. If the number of instances of each entity is now increased to two each, a possible combination satisfying the cardinality requirements of 1:1:1 may appear as follows:

X1 : Y1 : Z1
X2 : Y2 : Z2

The relation is maintained as 1:1:1 since for (X1,Y1) there is a single instance Z1; for (X1,Z1) there is a single instance Y1; for (Y1,Z1) there is a single instance X1, and so forth. Additionally in this relation, all binary relationships between X, Y and Z are also maintained as 1:1.



X1 : Y1 : Z1
X2 : Y2 : Z2
X1 : Y2 : Z3
X4 : Y4 : Z3

Table 1. 1:1:1 cardinality with all M:N binary relationships

Given this set of instance combinations, no other possible combinations of the identified entity instances can exist without violating the cardinality restrictions. For example, adding the tuple (X1, Y1, Z2) to the relation violates the ternary cardinality of 1:1:1, by creating 1:M between the pair of (X, Y) and the single entity Z, resulting in a ternary cardinality of 1:1:M.

If the number of instances for the three entities is now legitimately increased to three, it can be demonstrated that with certain ternary combinations, the binary relationships possible between pairwise entities begin to change from cardinalities of 1:1, to cardinalities of M:N. The following is a relation satisfying the 1:1:1 ternary cardinality:

X1 : Y1 : Z1
X2 : Y2 : Z2
X1 : Y2 : Z3

While this relation does not represent the full range of possibilities of the instances available, it does demonstrate that any of the binary relationships embedded in the ternary relationship tend towards a M:N basis. The binary relationship between entities X and Y is already established as M:N. Relationships X:Z and Y:Z are both 1:M. Yet the cardinality for the overall relation is maintained as 1:1:1.

If we now consider instances with no restriction on the number of each, it should be relatively easy to follow the logic that all binary relationships within the ternary relationship can be established as M:N. Table 1 shows a series of instances with a ternary relationship satisfying the 1:1:1 cardinality requirement.

Note also that all binary relationships within the ternary relationship (Table 1) have now been established as M:N.

A more realistic example helps to clarify this. The entities STUDENT, COURSE and TERM can be modeled as a ternary relationship. Example 1 shows a set of instances satisfying the ternary cardinality of 1:1:1. Additionally, the binary considerations in this relation are all M:N. Mike is associated with multiple courses and terms; Chemistry has multiple students and is offered in multiple terms; the spring term has multiple students and multiple courses.

STUDENT	COURSE	TERM
Mike	Physics	Fall
Mary	Chemistry	Winter
Mike	Chemistry	Spring
Kate	Biology	Spring

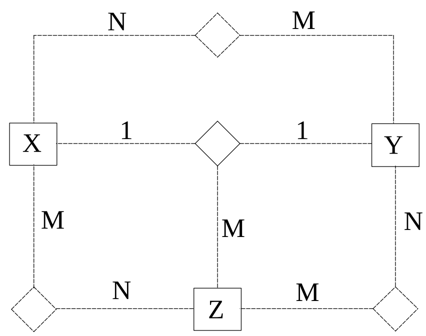
Example 1. 1:1:1 cardinality with all M:N binary relationships

3.2. Ternary Cardinality -- 1:1:M

Following from 3.1, above, we have demonstrated that a ternary relationship having 1:1:1 cardinality can have M:N binary cardinalities. It should follow, therefore, that demonstrating this phenomenon with a relation having 1:1:M cardinality is simpler since some binary relationships must already be established as 1:M. This is due to the minimum requirements of establishing the initial ternary relation. Consider the following relation:

X1 : Y1 : Z1
X1 : Y1 : Z2

This relation *minimally* shows the cardinality requirements of the ternary relationship (1:1:M)[1]. For a pair of X and Y entity instances (X1,Y1), there are two values of the entity Z (Z1 and Z2); for any pair of Y and Z, there is only one instance of X; and for any pair of X and Z instances, there is only one instance of Y. The binary relationships between X:Z and Y:Z are already 1:M. Following the arguments made in Section 3.1, all binary relationships in this ternary relationship can also be demonstrated to be M:N, allowing for a sufficient number of instances and



X1 : Y1 : Z1
X1 : Y1 : Z2
X2 : Y1 : Z3
X1 : Y2 : Z3

Table 2. 1:1:M cardinality with all M:N binary relationships

[1]This does not mean to imply that these instances are *required* for the relation. Only that this set of instances is the minimal set to fully demonstrate all functional dependencies under consideration.

no specific restriction on the cardinality between any specific pair. Continuing from the original relation above - we can add additional tuples to produce the diagram and instances as in Table 2.

The original cardinality of 1:1:M has been maintained, but now all embedded binary relationships are established as M:N. Example 2., below, demonstrates a set of real instances:

STUDENT	COURSE	TERM
Mike	Physics	Fall
Mike	Physics	Winter
Mary	Physics	Spring
Mike	Chemistry	Spring

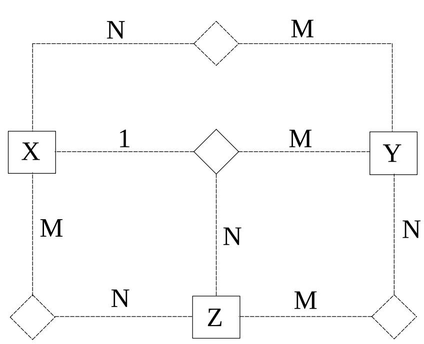
Example 2. 1:1:M cardinality with all M:N binary relationships

3.3. Ternary Cardinality -- 1:M:N

Again, a minimum set of instances sufficient to show the cardinality of the ternary relationship (1:M:N), also demonstrates the cardinalities of the individual embedded pairs[1] :

X1 : Y1 : Z1
X1 : Y1 : Z2
X1 : Y2 : Z1

In the first three tuples, which create a 1:M:N relation, X:Y is 1:M, X:Z is 1:M, Y:Z is M:N. By adding one additional instance (X2:Y2:Z2), the cardinalities of each binary pair are all changed to M:N without violating the 1:M:N cardinality requirement of the ternary as shown in Table 3.



X1 : Y1 : Z1
X1 : Y1 : Z2
X1 : Y2 : Z1
X2 : Y2 : Z2

Table 3. 1:M:N cardinality with all M:N binary relationships

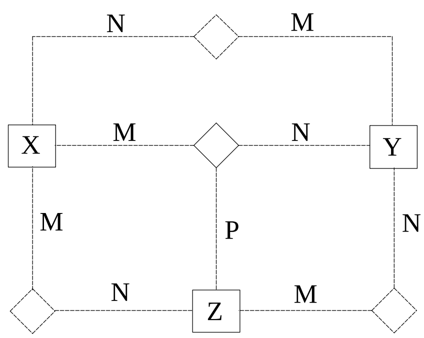
Example 3, below, offers a table of possible instances:

STUDENT	COURSE	TERM
Mike	Physics	Fall
Mike	Physics	Winter
Mike	Chemistry	Fall
Mary	Chemistry	Winter

Example 3. 1:M:N cardinality with all M:N binary relationships

3.4. Ternary Cardinality -- M:N:P

The final cardinality combination for a ternary relationship is that of M:N:P. Following the reasoning and proofs given in previous sections, it is easily demonstrated that the minimum set of instances required to show this ternary relationship also must show a M:N relationship between any pair of instances. For example:



X1 : Y1 : Z1
X1 : Y1 : Z2
X1 : Y2 : Z1
X2 : Y1 : Z1

Table 4. M:N:P cardinality with all M:N binary relationships

This relation is the *minimal* requirement to show the M:N:P cardinality[3]. We note that X:Y is M:N, X:Z is M:N and Y:Z is M:N. Through this relation, then, we have established again that all binary relationships must be M:N given no restriction on the addition of tuples. An example of instances is as shown in Example 4:

STUDENT	COURSE	TERM
Mike	Physics	Fall
Mike	Physics	Winter
Mike	Chemistry	Fall
Mary	Chemistry	Winter

Example 4. M:N:P cardinality with all M:N binary relationships

3.5. Summary of Implicit Binary Cardinalities

Following the heuristics demonstrated in Sections 3.1 -- 3.4, we can conclude that the specification of a ternary cardinality constrains only the combination of entity instances, and not the cardinalities of the embedded binary relationships. Further, given that there are no explicit restrictions on the number of instances that can occur, we must allow that the possible implicit relationships between any two entities will always be M:N. This has important ramifications in considering possible decomposition strategies. We can therefore establish the **Implicit Binary Cardinality (IBC)** rule:

> **In any given ternary relationship, regardless of ternary cardinality, the implicit cardinalities between any two entities must be considered M:N, provided that there are no explicit restrictions on the number of instances that can occur.**

4. SINGLE EXPLICIT BINARY IMPOSITION

In the previous sections, we considered implicit binary cardinalities in ternary relationships where there is no explicit restriction on the number, or combination, of instances that can occur. The next issue is to consider the explicit restrictions resulting from binary relationships imposed on the various ternary combinations. How does this imposition affect possible instance combinations and possible resultant binary combinations within the ternary? We must keep in mind that a ternary relationship is still the basic determination of these structures including the implicit relationships as established in Section 3, above.

4.1. Binary Imposition

Consider a ternary relationship R(X, Y, Z), having cardinality M:N:P. Additionally, consider that a binary relationship of 1:1 is required to be imposed between entities X and Z. Table 4 (Sect. 3.4) demonstrates a minimal set of tuples that completes the requirements of the M:N:P ternary relationship. That table is reproduced as follows:

X1 : Y1 : Z1
X1 : Y1 : Z2
X1 : Y2 : Z1
X2 : Y1 : Z1

From this set of instances, we can immediately determine that the requirements of the ternary relationship are in conflict with the requirements of the binary relationship. The ternary requires the unique pair of instances X1:Y1 to have multiple instances of entity Z. Inherent in this structure is the requirement for both entities X and Y to individually have a relationship with Z that is 1:M.

Logically then, we can not comply with the establishment of the binary requirement of 1:1 between X and Z.

Another example may help illustrate this finding. Can we impose a binary relationship with cardinality M:1, between X and Y that are parts of a ternary relationship having 1:M:N (X:Y:Z) cardinality? The ternary relationship (1:M:N) can be minimally represented as follows (also Table 3):

X1 : Y1 : Z1
X1 : Y1 : Z2
X1 : Y2 : Z1

Since this is a *minimal* representation of the 1:M:N cardinality, we can not remove any tuple. Note that the relationship X:Y is of cardinality 1:M. But the imposition of the binary requires that the relationship between X and Y be M:1. The binary imposition is therefore disallowed because the requirement for a single instance of Y for each instance of X has already been violated during construction of the ternary relationship. In other words, the cardinality of entity Y in the ternary relationship could not be reduced to accommodate the binary imposition. This example also supports our statements from Section 1 regarding disallowed ERD's used in previous research papers. The imposed binary relationship is in conflict with the implicit requirements of the ternary relationship (see also Table 7, below).

We can now establish a primary rule for testing potential combinations of ternary and binary relationships. The **Explicit Binary Permission (EBP)** Rule states:

For any given ternary relationship, a binary relationship cannot be imposed where the binary cardinality is less than the cardinality specified by the ternary, for any specific entity.

For example, if the ternary cardinality is defined as M:N:P, then a binary cardinality containing a 1:M or 1:1 cannot be imposed between any two entities because the binary cardinalities would be less than those specified in the ternary relationship. The two requirements would be in conflict.

Tables 5 through 8 identify all enumerations of binary imposition allowance and non-allowance for all ternary cardinality combinations:

Any Cardinality of binary relationship can be imposed on 1:1:1 ternary relationship

Table 5. Allowed and Disallowed Binary Impositions in 1:1:1 Ternary Relationships

X	Y	Z	allowed?
M	N		Y
M	1		Y
1	1		N
1	M		N
M		N	Y
M		1	Y
1		1	N
1		M	N
	M	M	Y
	M	1	Y
	1	1	Y
	1	M	Y

Table 6. Allowed and Disallowed Binary Impositions in M:1:1 Ternary Relationships

X	Y	Z	allowed?
M	N		Y
M	1		N
1	1		N
1	M		N
M		N	Y
M		1	Y
1		1	N
1		M	N
	M	M	Y
	M	1	Y
	1	1	N
	1	M	N

Table 7. Allowed and Disallowed Binary Impositions in M:N:1 Ternary Relationships

Only binary cardinalities of M:N are allowed for imposition in M:N:P ternary relationships. (Imposition is redundant since the ternary relationship implicitly establishes the binary cardinality constraints).

Table 8. Allowed and Disallowed Binary Impositions in M:N:P Ternary Relationships

The EBP rule suggests that in a given ternary relationship, not every binary cardinality is allowed because of the constraints of the ternary. Therefore when we impose a binary relationship in a given ternary, we must consider whether the binary relationship is allowed or not, by comparing the cardinalities of the two.

4.2. Overrides of Implicit Binary Cardinality Rule

Next, we consider the imposition of binary relationships on ternaries that do not violate the EBP rule established in Section 4.1. Consider a ternary relationship having 1:1:1 cardinality. In Section 3.5, we have established that the IBC rule implies that any binary relationship within this ternary can be considered M:N. By imposing a binary relationship between the two entities X and Y of 1:1, for example, we must effectively restrict the number of possible instance combinations to those abiding by the binary specifications. For example, the following relation adheres to the 1:1:1 ternary requirements:

X1 : Y1 : Z1
X2 : Y2 : Z2

In Section 3.1, above, we were able to add a further tuple without violating the ternary cardinality as follows:

X1 : Y1 : Z1
X2 : Y2 : Z2
X1 : Y2 : Z3

Now, however, this does not comply with the required X:Y binary requirements of 1:1, because we are attempting to create a 1:M relationship between X and Y. Therefore this combination of instances must be disallowed. The imposition of the binary is effectively restricting the possible addition of tuples to those not violating the 1:1 relationship between X and Y. The cardinality of the relationship between the two entities is therefore controlled by the binary relationship and is no longer unrestricted (or M:N) as determined in Section 3.

Following the reasoning given here, we can establish an **Implicit Binary Override (IBO)** Rule:

Given the imposition of a *permitted* binary relationship on a ternary relationship, the cardinality of the binary relationship determines the final binary cardinality between the two entities involved.

This rule states that once a binary relationship is allowed within a ternary relationship, the cardinality of the binary relationship overrides the implicit binary cardinality of M:N derived from the ternary relationship. We reiterate that this IBO rule cannot be applied to invalid binary relationships disallowed by the EBP rule.

As an example, if we consider a ternary relationship, X, Y, Z with cardinality of M:1:1. We have previously (Section 3.2) established that the implicit cardinality between any *pair* of entities (e.g., between X and Y) is M:N. If we now impose a binary relationship between X and Y with cardinality of M:1 (which is allowed by the EBP rule), then the relationship between X and Y now has cardinality of M:1 (the IBO rule), and not M:N as established by the IBC rule in Sections 3.5.

4.3. Results of Single Binary Imposition

In this section we show that the imposition of one SCB relationship, (allowed by the EBP rule), affects the cardinality of other embedded binary relationships. We first show the summary tables, followed by selected proofs using functional dependencies. All ternary relationships having such impositions can be similarly proved.

Tables 9 through 12 demonstrate all allowed binary impositions together with resulting implicit cardinalities, for all ternary cardinality combinations (note that imposed binary cardinalities of M:N are not shown since it is understood that they are *implicitly* required without an *explicit* condition specifying otherwise -- IBC rule). The tables represent all allowable impositions of binary relationships on the specified ternary relationships according to the EBP rule.

	imposed binary cardinality	resultant implicit binary cardinalities
9(a)	X:Y is M:1	X:Y is M:1 X:Z is M:1 Y:Z is M:N
9(b)	X:Y is 1:M	X:Y is 1:M X:Z is M:N Y:Z is M:1
9(c)	X:Y is 1:1	X:Y is 1:1 X:Z is M:1 Y:Z is M:1
9(d)	All Other Binary	Due to the symmetry of the ternary relationship, other binary impositions can be re-created using the structures above.

Table 9. Results of Binary Imposition on 1:1:1

	imposed binary cardinality	resultant implicit binary cardinalities
10(a)	X:Y is M:1	X:Y is M:1 X:Z is M:1 Y:Z is M:N
10(b)	X:Z is M:1	X:Y is M:1 X:Z is M:1 Y:Z is M:N
10(c)	Y:Z is M:1	X:Y is M:N X:Z is M:N Y:Z is M:1
10(d)	Y:Z is 1:M	X:Y is M:N X:Z is M:N Y:Z is 1:M
10(e)	Y:Z is 1:1	X:Y is M:N X:Z is M:N Y:Z is 1:1

Table 10. Results of Binary Imposition on M : 1 : 1

	imposed binary cardinality	resultant implicit binary cardinalities
11(a)	X:Z is M:1	X:Y is M:N X:Z is M:1 Y:Z is M:N
11(b)	Y:Z is M:1	X:Y is M:N X:Z is M:N Y:Z is M:1

Table 11. Results of Binary Imposition on M : N : 1

Only M:N can be established. No binary imposition other than M:N is allowed. (See also Table 8.)

Table 12. Results of Binary Imposition on M:N:P

The analysis shows that imposing one binary relationship on a ternary relationship can also change the cardinalities of other implicit binary relationships. Note that in Tables 9(a, b, & c), and 10(a & b), more than one binary cardinality has been changed by the imposition of the single binary relationship. For example, in Table 10(a), imposing M:1 between X:Y (IBO rule) also changes the cardinality of X:Z from M:N (IBC rule) to M:1.

All structures enumerated in Tables 9 through 12 can be supported with proofs from functional dependency (FD) analysis. Two examples follow:

Proof of 9(a):

Consider the ternary relationship, cardinality 1:1:1. We impose a binary constraint of M:1 between X:Y. All functional dependencies are as follows:

$XY \rightarrow Z$

$YZ \rightarrow X$

$XZ \rightarrow Y$

$X \rightarrow Y$

Derived from this, the minimal cover is:

$X \rightarrow Z$ $YZ \rightarrow X$ $X \rightarrow Y$

We note that a FD, $A \rightarrow B$, implies M:1 between A and B, unless there is an explicit FD, $B \rightarrow A$, when the cardinality would be 1:1. This indicates that X:Z = M:1 since there is no FD $Z \rightarrow X$; Y:Z = M:N since there is no explicit dependency either way; X:Y = M:1 since there is no FD $Y \rightarrow X$. These three resultant cardinalities, following the imposition of the X:Y, M:1 relationship, are shown in Table 9(a).

Proof of 10(a):

The implicit functional dependencies (FD's) involved in a ternary relationship of cardinality M:1:1 are as follows (Teorey, 1990):

$XY \rightarrow Z$

$XZ \rightarrow Y$

But imposing M:1 between X:Y introduces another functional dependency $X \rightarrow Y$. With the new functional dependency, the original set is no longer minimal. Removing the redundancies, we can produce the minimal set as follows:

$X \rightarrow Y$ and $X \rightarrow Z$

Hence the above FD $X \rightarrow Y$ implies M:1 between X and Y, and FD $X \rightarrow Z$ implies M;1 between X and Z, since there are no explicit FD's $Y \rightarrow X$ and $Z \rightarrow X$. Also since no dependencies can be identified between Y and Z, we can conclude this relationship to be M:N. The embedded binary relationships then have a set of cardinalities proven to be as follows:

X:Y is M:1 X:Z is M:1 Y:Z is M:N

This set of relationships determined by analysis of functional dependency is identified in 10(a) above.

5. SECONDARY EXPLICIT BINARY IMPOSITION

The imposition of a single binary relationship on a ternary relationship may have varying repercussions on the rest of a relationship. By examining the tables provided in Section 4.2 (Tables 9 through 12), we can see that the imposition of binary relationships on certain ternary cardinalities sometimes requires that other binary relationships are also constrained. For example, the imposition of a binary constraint of 1:1 on a ternary cardinality of 1:1:1 requires that the cardinality between the remaining binary relationships both be M:1 (see Table 9(c)). Other combinations have various required cardinalities and unconstrained relationships, except that of a ternary relationship of M:N:P. Since this cardinality does not allow binary imposition (other than M:N), according to the EBP rule, the structure must remain constant and all implicit binary relationships are unconstrained.

Having described the resultant cardinalities following the imposition of a single binary constraint, the next logical step is to investigate each structure to test whether it allows further binary impositions. Again, using a generic relation R(X, Y, Z), we extend Tables 9 -- 12 to investigate alternative configurations available following the imposition of a second SCB relationship. Tables 13 -- 16, Sections 5.1 -- 5.4, enumerate all possible secondary impositions and the resultant ternary structures. As in Section 4.3, where we established a proof of the implicit cardinality structure from functional dependencies, Section 5.5 provides example proofs of cardinality structures derived from multiple binary impositions.

5.1. Ternary cardinality 1:1:1

We provide an example to explain the dynamics involved when multiple impositions are considered. Table 13, which is an extension of Table 9, shows only those initial binary impositions that allow secondary impositions, together with the resultant cardinalities derived from the secondary impositions.

Consider the relation R(X, Y, Z) with cardinality of 1:1:1. Following the imposition of a binary relationship between X and Y of cardinality M:1, the binary relation X:Z must become M:1 and Y:Z can remain M:N (Table 9(a)).

Since the binary pair Y:Z is still unconstrained, we can consider a possible additional imposition between this pair, provided it is allowed by the EBP rule and does not impinge on the previous, *explicitly* established cardinalities (both the binary and ternary).

Logically, it is not possible to impose a binary constraint on Y:Z of 1:1, since this would require the cardinality between X:Y to conform to the structure shown in 9(c) or 1:M. This would be in conflict with the previously established cardinality constraint between X and Y of M:1. Since the binary relationship of X:Y was initially established as M:1, the 1:1 cardinality between Y:Z is disallowed.

Likewise, the same argument holds for the imposition of a binary cardinality of M:1 between Y and Z. Attempting to impose this cardinality between Y and Z, requires a 1:1 relationship between X:Y that conflicts with the M:1 relationship established by the first imposition between X:Y. The secondary imposition of M:1 between Y and Z is also disallowed.

The remaining possibility of 1:M between Y:Z is allowed, but we have an additional implication. Consider that X:Y was *explicitly* imposed as M:1, and Y:Z has now been *explicitly* imposed as 1:M. Following the initial imposition between X and Y the relationship X:Z was *implicitly* derived due to the imposition of the X:Y constraint. With the additional imposition of the Y:Z constraint, the relationship X:Z is now implicitly required to adopt a binary cardinality derived from the dual imposition. This implicitly derived cardinality *must* be 1:1. The resultant structure caused by the dual imposition, is identical to that derived in Table 9(c), where all three relationships are mandated (or Table 13b, where the 1:1 imposition between X and Z is considered as the secondary imposition, fixing the binary relationship Y:X as 1:M).

	initial binary	result	additional binary	result
13(a)	X:Y is M:1	X:Y is M:1 X:Z is M:1 Y:Z is M:N	Y:Z is 1:M	X:Y is M:1 X:Z is 1:1 Y:Z is 1:M
13(b)	X:Y is M:1	X:Y is M:1 X:Z is M:1 Y:Z is M:N	X:Z is 1:1	X:Y is M:1 X:Z is 1:1 Y:Z is 1:M

Table 13. 1:1:1 with multiple binary impositions[2]

5.2. Ternary Cardinality M:1:1

Due to the equal symmetry of the structure in a 1:1:1, the resultant structure derived in Section 5.1 was the same, regardless of which imposition was made first, or in which order. In the case of M:1:1, the situation is more complex. We can consider this case in two distinct sections. The first is if the initial binary imposition is between X:Y (or by symmetry, X:Z), and the second if the initial binary imposition is between Y:Z.

(a) According to the EBP rule, the only imposition allowed between X:Y (or X:Z) is M:1 (see Table 6). Either of these sets up a cardinality structure as previously described in Table 10(a & b), with X:Z and X:Y both being M:1. Y:Z is unchanged as M:N and therefore subject to

[2] 1:1 between Y:Z, M:1 between Y:Z and 1:M between X:Z are invalid as additional impositions due to conflict with cardinality requirements of initial binary imposition.

	initial binary	result	secondary binary	result
14(a)	X:Y is M:1 (also X:Z is M:1)	X:Y is M:1 X:Z is M:1 Y:Z is M:N	Y:Z is 1:1	X:Y is M:1 X:Z is M:1 Y:Z is 1:1
14(b)	X:Y is M:1 (also X:Z is M:1)	X:Y is M:1 X:Z is M:1 Y:Z is M:N	Y:Z is M:1	X:Y is M:1 X:Z is M:1 Y:Z is M:1
14(c)	X:Y is M:1 (also X:Z is M:1)	X:Y is M:1 X:Z is M:1 Y:Z is M:N	Y:Z is 1:M	X:Y is M:1 X:Z is M:1 Y:Z is 1:M

Table 14. M:1:1 with multiple binary imposition

further imposition. Unlike the considerations in the 1:1:1 cardinality, according to the EBP rule, this cardinality allows the imposition of any binary cardinality between Y:Z. The potential combinations are as shown in Table 14.

(b). The second alternative, where the initial imposition of the binary is between Y:Z, is closely related to the initial alternative. In this situation, we can consider various options in the initial imposition, but the secondary choices for imposition are limited due to the first. From the analysis in Table 10 (c),(d) & (e) (also left side of Table 15, below),we can see that regardless of the cardinality imposed between Y:Z that the remaining binary cardinalities remain as M:N. That is, there is no implicit constraint on either, caused by the initial imposition. In considering further impositions, we must also remember the implicit requirements of the ternary itself. Since both remaining relationships (X:Y and X:Z) are both M:1 within the ternary, the potential binary impositions are restricted according to the EBP rule. Due to the EBP rule, 1:1 impositions are disallowed, but M:1 constraints are allowed. The potential combinations are detailed in Table 15.

	initial binary	result	secondary binary	result
15(a)	Y:Z is 1:1	X:Y is M:N X:Z is M:N Y:Z is 1:1	X:Y is M:1 (also X:Z is M:1)	X:Y is M:1 X:Z is M:1 Y:Z is 1:1
15(d)	Y:Z is M:1	X:Y is M:N X:Z is M:N Y:Z is M:1	X:Y is M:1 (also X:Z is M:1)	X:Y is M:1 X:Z is M:1 Y:Z is M:1
15(c)	Y:Z is 1:M	X:Y is M:N X:Z is M:N Y:Z is 1:M	X:Y is M:1 (also X:Z is M:1)	X:Y is M:1 X:Z is M:1 Y:Z is 1:M

Table 15. M:1:1 with multiple binary imposition[3]

[3] 1:1 and 1:M between X:Y (or X:Z) are invalid as additional impositions due to EBP rule.

5.3. Ternary Cardinality M:N:1

In this final ternary cardinality that allows explicit imposition of a binary relationship, the initial explicit imposition must be M:1 applied to X:Z or Y:Z (according to the EBP rule, Table 4(a)). We note that the imposition of the M:1 constraint does not affect the M:N cardinalities of the remaining two binary relationships. However, no further imposition can be made between X:Y due to the EBP rule (the ternary relationship requires M:N between X and Y). The single remaining combination between Y and Z, is likewise governed by the EBP rule but does allow the imposition of a M:1 without any conflict or impact on the previously established structure. The Table of possibilities for the ternary relationship of M:N:1 is as follows:

	initial binary	result	secondary binary	result
16(a)	X:Z is M:1 (or Y:Z is M:1)	X:Y is M:N X:Z is M:1 Y:Z is M:N	Y:Z is M:1 (or X:Z is M:1)	X:Y is M:N X:Z is M:1 Y:Z is M:1

Table 16. M:N:1 with multiple binary imposition[4]

5.4. Ternary Cardinality M:N:P

Since all binary pairs within the ternary are required to be M:N (unconstrained), the EBP rule does not allow for the imposition of any explicit binary cardinalities. There is, therefore, no consideration as to initial or secondary impositions for this cardinality.

5.5. Proof of Multiple SCB Relationship Imposition

As with the imposition of the single binary relationship on any specific ternary relationship, the resulting cardinality combination of imposing a second binary relationship can be proven with functional dependency analysis.

Consider a ternary relationship with cardinality M:1:1, with the binary imposition of Y:Z having cardinality of 1:1. The dependencies identified from these criteria are as follows:

$XZ \rightarrow Y$

$XY \rightarrow Z$

$Y \rightarrow Z$

$Z \rightarrow Y$

[4] 1:1 and 1:M between Y:Z are invalid as additional impositions due to EBP rule.

Since the only binary relationships that can be identified from these dependencies are those between Y and Z, we can conclude that the binary cardinalities within this structure are:

X:Y = M:N   X:Z = M:N   Y:Z = 1:1   (Table 10(e))

The imposition of a secondary binary relationship between X:Y of cardinality M:1, creates an additional dependency of $X \rightarrow Y$. If we add this to the original dependency set we can derive the following complete dependency set:

$XZ \rightarrow Y$
$XY \rightarrow Z$
$Y \rightarrow Z$
$Z \rightarrow Y$
$X \rightarrow Y$
$X \rightarrow Z$

Due to the identification of the dependencies between X:Y, X:Z and Y:Z , we can conclude the following embedded cardinalities:

X:Y = M:1   X:Z = M:1   Y:Z = 1:1   (Table 15(a))

All structures derived from the imposition of secondary binary relationships (Table 13 through 16) can be likewise validated using functional dependency analysis.

6. DECOMPOSITION STRATEGIES

In this section we briefly discuss the application of the binary imposition rules to the decomposition of ternary relationships. This is not intended to be a complete analysis of this topic, but an introduction, demonstrating the potential relevance of the rule set.

It is known that a ternary relationship can be decomposed into two binary relationships provided that a 1:1 relationship exists between any two entities involved in the ternary relationship (Elmasri & Navathe, 1989). We note, however, that this rule does not specify the implicitness or explicitness of the 1:1 binary cardinality with respect to the ternary relationship. Neither does the observation identify the ternary cardinalities on which the 1:1 binary cardinality can be imposed. According to our EBP rule, 1:1 is only allowed in 1:1:1 and 1:1:M ternary cardinalities (Table 5 & Table 6). Ternary cardinalities of M:N:P and M:N:1 can never have 1:1 relationships between any of two entities involved (Table 7 and Table 8). This implies that the rule stated by Elmasri & Navathe is not generalizable to all ternary relationships.

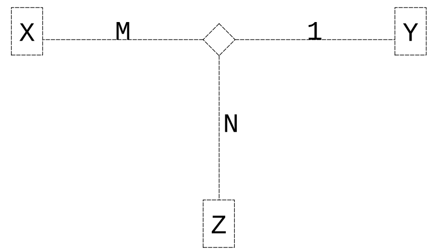


Figure 7. Ternary Relationship
with M:1:N Cardinality

If we have an analysis that imposes a binary relationship of 1:M on a ternary, then a further set of considerations arises from the possibility of decomposing this ternary relationship.

Figure 7 shows a ternary relationship having cardinality of M:1:N. Without an explicitly imposed binary relationship the cardinality between X and Y is M:N (IBC rule, Sect. 3.5).

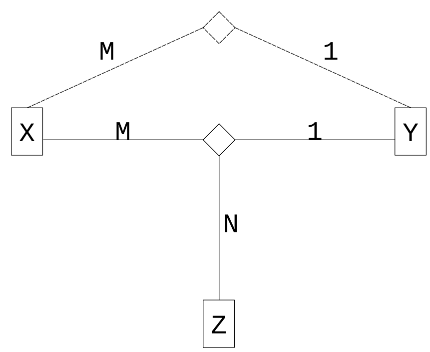


Figure 8. M:1:N with Imposed
M:1 Binary Relationship

Figure 8 shows the imposition of a binary relationship between X and Y with cardinality of M:1 (allowed by EBP rule, Sect. 4.1). We have now changed the actual relationship between X and Y to agree with the binary so the specific relationship between X and Y is now M:1 (Table 11(a),

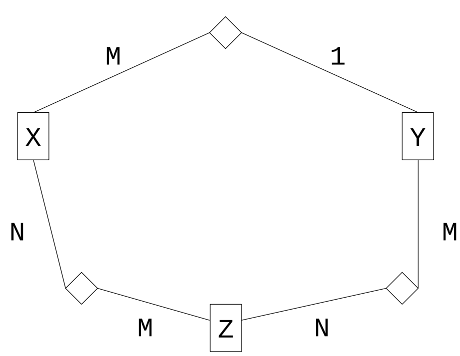


Figure 9. Binary
Representation of Figure 8

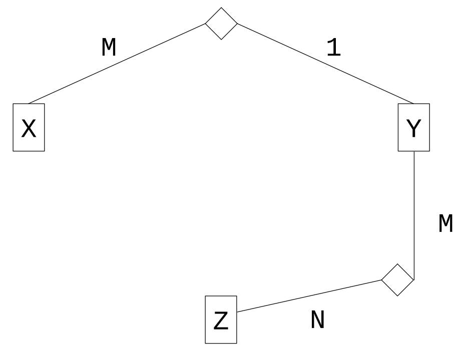


Figure 10. Lossy Decomposition
Of Figure 9

Sect. 4.2).

Since the implicit cardinalities between XZ and YZ are both M:N (IBC rule), we can view the ternary relationship as three binary relationships (Figure 9). This consideration does not negate the requirements of the ternary inclusion constraints.

The question is now posed; can the ternary relationship be losslessly decomposed to a simpler format? If we consider the decomposition as in Figure 10, a lossless decomposition cannot be guaranteed.

In Figure 10, we see that the relationship Z:Y is M:N, that is, for each instance of Z we have multiple instances of Y. Further, the directional relationship Y:X is likewise 1:N. If we then consider whether we can associate all correct instances of Z with the correct instances of X, it is possible to develop spurious tuples from making all possible associations through Y, which may not have been present in the original instance set. The following example demonstrates this:

X1 : Y1 : Z1
X1 : Y1 : Z2
X2 : Y1 : Z1
X3 : Y3 : Z1

This relation, demonstrates a ternary relationship adhering to the required cardinality of M:1:N, as well as demonstrating a specific binary cardinality of M:1 between X:Y. Thus the cardinality requirements are met, as well as the rules of ternary and binary combination established earlier. Following the strategy in Figure 10, this relation can be decomposed into the following two tables:

X1 : Y1	Y1 : Z1
X2 : Y1	Y1 : Z2
X3 : Y3	Y3 : Z1

The decomposition maintains the cardinalities of M:1 between X and Y, and M:N between Y and Z. If the tables are now recomposed using all possible combinations of X and Z through Y, we can develop a spurious tuple X2:Y1:Z2 that was not part of the original relation.

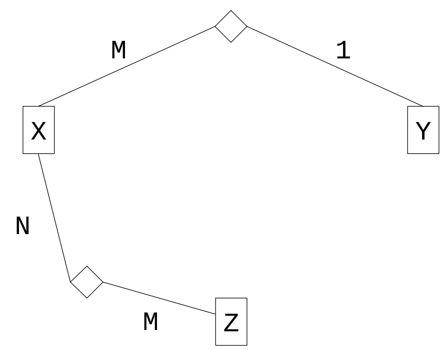


Figure 11. Lossless Decomposition
of Figure 9

Alternatively, if we decompose the table with entity X as the common entity (diagrammatically shown in Figure 11), we arrive at the following set of tables:

X1 : Y1	X1 : Z1
X2 : Y1	X1 : Z2
X3 : Y3	X2 : Z1
	X3 : Z1

If we now recompose these tables we note that the original table is recreated, indicating a lossless decomposition. This decomposition strategy is also supported by the functional dependency analysis shown in Section 4.3 where we developed the dependency of both Y and Z on entity X.

This section has demonstrated that decomposition strategies are available in situations other than when a 1:1 cardinality exists in a ternary relationship. When considering the use of ternary relationship modeling in ER diagrams we must consider a number of factors to decide whether the ternary can be decomposed to two binary relationships. Elmasri (1989) established a criterion for decomposition when a 1:1 binary relationship exists. Here we have identified the circumstances where this condition *can* exist, as well as going beyond these considerations to demonstrate other possibilities for decomposition of ternary relationships, specifically where a 1:M exists.

7. SUMMARY

In this paper we have progressively considered ternary relationships and the potential for combining them with binary relationship constraints. Initially we identified the implicit cardinalities of the binary combinations embedded in ternary relationships and established the Implicit Binary Cardinality (IBC) rule. Next we identified which binary cardinalities could be imposed on which ternary relationships, establishing the Explicit Binary Permission (EBP) rule. The final part of the analysis consisted of demonstrating how the imposition of binary relationships on ternary structures, alters the implicit binary relationships between the entities. This established the Implicit Binary Override (IBO) rule. This rule set is particularly important since the theory behind the co-existence of binary relationships within ternary relationships has not been formalized. We have found instances where our findings presented here correct examples offered in previous works (e.g., Jajodia et al. (1983), p.622; Markowitz & Shoshani (1989), p.432; Markowitz Shoshani (1992), p.426). We believe this rule set provides parameters for ternary relationship specification during ER modeling. Previously, no theoretical basis has been offered to explain the dynamics implicit in ternary relationships. Our work now provides a basis for both practitioners, to incorporate into their modeling structures, as well as educators, to explain the formalities of ternary structures and combinations of such with binary relationships.

Section 6 has demonstrated how the findings and rules established in this paper, are important to any considerations of ternary relationship decomposition. We have demonstrated that given the presence of certain binary relationships imposed on ternary relationships, the ternary can be decomposed to two binary relationships.

We have provided a framework which will allow users of ER models to verify the accuracy of their use of ternary relationships. Our rules also provide a means for identifying the potential for decomposing the ternary relationship to a simpler format and additionally, the means of ensuring the correct decomposition. We believe these findings to be important to analysts and designers seeking assistance in accurately modeling ternary relationships when using ER models. One aspect of ternary relationship not addressed is the question of the initial identification of ternary relationships. This question must be answered from the point of view of the semantics or situational dynamics of a particular problem domain. Since this paper has addressed the analysis of the logic inherent to *predetermined* ternary relationships, we leave the question of specifically *when to use a ternary relationship* to future analysis.

REFERENCES

Batini, C., Ceri, S., Navathe, S. *Conceptual Database Design*. Benjamin/Cummings, Redwood City, CA. 1992.

Chen, P.P. (1976) The entity-relationship model: towards a unified view of data. *ACM Transactions on Database Systems*, 1(1), pp. 9 -- 36.

Chung, I., Nakamura, F., Chen, P. (1981) A decomposition of relations using the entity-relationship approach. In *Proceedings of the Second International Conference on Entity-Relationship Approach (Washington, D.C.)*. ER Institute, Saugus, CA. 1981, 151 -- 173.

Elmasri, R., and Navathe, S. *Fundamentals of Database Systems*. Benjamin/Cummings, Redwood City, CA. 1989.

Hawryszkiewycz, I.T. *Database Analysis and Design*, 2nd Ed. Macmillan Publishing Co., 1991.

Jajodia, S., Ng, P., Springsteel, F. (1983) The problem of equivalence for entity-relationship diagrams. *IEEE Transactions on Software Engineering,* v. SE-9(5), September, 1983, 617 -- 630.

Ling, T-W. (1985a) A normal form for entity-relationship diagrams. In *Proceedings of 4th International Conference on the Entity-Relationship Approach (Chicago).* IEEE Computer Society Press, Silver Springs, MD., 1985, pp. 24 -- 35.

Ling, T-W. (1985b) An analysis of multivalued and join dependencies based on the Entity-Relationship approach. *Data and Knowledge Engineering*, v.1, pp. 253 -- 271.

Markowitz, V.M. & Shoshani, A. (1989) On the correctness of representing extended entity-relationship structures in the relational model. In *Proceedings of 1989 ACM SIGMOD*, Portland, OR., May 31 -- June 2, 1989, pp. 430 -- 439.

Markowitz, V.M. & Shoshani, A. (1992) Representing Extended Entity-Relationship Structures in Relational Databases: a Modular Approach. *ACM Transactions on Database Systems*, v.17(3), September, 1992, pp. 423 -- 464.

Teorey, T.J. *Database Modeling and Design: The E-R Approach*. Morgan-Kauffman, 1990.

Teorey, T.J., Yang, D., Fry, J.P. (1986) A logical design methodology for relational databases using the extended entity-relationship model. *Computing Surveys*, 18:12, June, pp. 197 -- 222.